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CONTRIBUTION OF ANTISYMMETRIC AND SYMMETRIC WAVES TO THE REFLEC--ETC(U)

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20. Abstract (continued)

assumed to occur separately. It is shown that the relative strength of the two wave types, as measured by the cross-sectional average of the square of the particle displacement, is strongly dependent on coincidence between the trace speed of the incident wave in the fluid and the free wave speed of the pertinent wave type in the plate. Away from resonances, the energy densities of the two wave types appear to be about equal. This leads to the conclusion that the contribution of symmetric waves to the reflection of sound by a plate should be taken into account. ←

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CONTRIBUTION OF ANTISYMMETRIC AND SYMMETRIC WAVES TO THE REFLECTION OF SOUND IN A FLUID BY A THICK, HOMOGENEOUS PLATE

INTRODUCTION

In a recent report Rudgers [1] describes the acoustical behavior of thick, fluid-loaded plates. Since exact elasticity theory leads to considerable mathematical complexity and does not lend itself, in general, to easy physical interpretation, the analysis in that report is based on the Timoshenko-Mindlin thick-plate theory [2]. The principal contribution of the latter theory is the introduction into classical plate theory of transverse shear, for the case of bending, or flexural, waves. The approximation inherent in this theory is represented by an effective shear modulus. This modulus is fixed by Mindlin in such a way that the wave speed approaches the Rayleigh wave speed at high frequencies.

Further analysis of this effective shear modulus is discussed in a report by Dubbelday [3], where it is shown that one may derive the correction factor for the shear modulus by comparing the approximate theory with the exact elasticity theory. Moreover, a similar development may be applied to extensional waves. Both types of waves, the flexural and the extensional, can be considered as the low-frequency representatives of zero order Lamb waves, the antisymmetric corresponding to flexural waves and the symmetric corresponding to extensional waves. In Ref. [3], a thick-plate formulation is developed for the symmetric waves, and a correction factor is defined for the shear modulus that is analogous to the correction factor in Timoshenko-Mindlin theory.

The analogous behavior of the two wave types prompted an investigation of the relative contribution of antisymmetric and symmetric waves to the plate-fluid interactions studied by Rudgers [1]. In the present report, the role of antisymmetric and symmetric waves is analyzed for the case where a plane wave is reflected by a homogeneous, isotropic plate loaded by a fluid on one side. First, the problem is stated in mathematical form for the more general case where a fluid is situated on each side of the plate. Next, the reflection coefficient is evaluated for the case where the plate is fluid loaded on one side only, the fluid on the other side becoming a vacuum. The reflection is interpreted as being due to the cooperation of an antisymmetric wave and a symmetric wave. The reflection coefficient can be described in terms of a "structural response function", as defined by Rudgers [4]. The structural response function of the two types of waves in combination is related to the response functions of the two types separately as an effective impedance given in terms of the values of two parallel impedances.

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This study differs in several aspects from work on the subject of wave reflection by plates by Lyamshev [5] and Graff *et al.* [6]. Lyamshev considers reflection due to flexural and extensional waves from the point of view of thin-plate theory. Graff *et al.* modify the Lyamshev approach by replacing the flexural wave of thin-plate theory by that arising from Timoshenko-Mindlin thick-plate theory. The extensional waves, however, are still modeled by thin-plate theory. In the present study, both flexural and extensional waves are modeled by thick-plate equations and thus are corrected for the influence of effective shear. As a consequence, the result can be applied for high frequency also, and moreover the effect of the first-order antisymmetric and symmetric waves is included. Also in the present report, the relative contribution of the two wave types to the reflection is shown; this is not discussed in the two references mentioned above.

A quantitative measure for the relative "strength" of the two wave types is obtained by computing the ratio of the cross-sectional average of the square of the particle displacement amplitudes for the symmetric and antisymmetric waves. This ratio equals the ratio of the average kinetic energy densities associated with the two wave types. When this ratio is plotted as a function of the angle of incidence of the incoming wave, it shows a resonance phenomenon. Whenever the trace speed of the wave in the fluid equals the wave speed of a free wave in the plate of a given type, the amplitude of that given wave type dominates over the other. Away from resonances the ratio of the energies of the two wave types is about unity.

MATHEMATICAL FORMULATION

The geometry of a plane wave in a fluid interacting with a flat plate is shown in Fig. 1. The incident wave, with wave number k_0 in a fluid with acoustic wave speed c_0 and density ρ_0 , impinges on a plate at an angle θ measured with respect to the normal to the plate. The density of the plate is ρ_s and its thickness is $2d$ (comparable to the thickness h as used by some authors; e.g., Ref. [2]). The wave number of straight-crested waves in the plate is k , and the phase speed is c . For the sake of generality, it is assumed that the opposite side of the plate is in contact with a fluid of density ρ' and phase speed c' . The transmitted wave has a wave number k' and an angle of transmission θ' .

The two-dimensional wave vectors are represented by the expressions

$$\vec{k}_0 = k_0 \sin \theta \vec{i} - k_0 \cos \theta \vec{k} \quad (1)$$

for the incident wave,

$$\vec{k}_0 = k_0 \sin \theta \vec{i} + k_0 \cos \theta \vec{k} \quad (2)$$

for the reflected wave, and

$$\vec{k}' = k' \sin \theta' \vec{i} - k' \cos \theta' \vec{k} \quad (3)$$

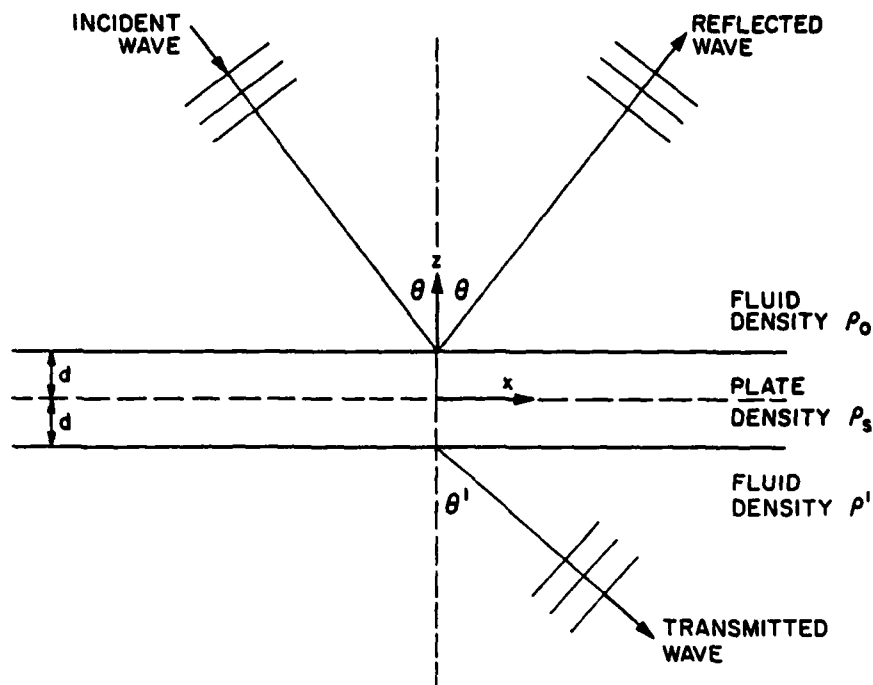


Fig. 1 - Geometry of fluid-loaded plate

for the transmitted wave. The unit vector \vec{i} is parallel to the faces of the plate, the unit vector \vec{k} is perpendicular to the faces of the plate. It is assumed that these three waves have a harmonic dependence on time and space coordinates. The assumed time-dependence $\exp(i\omega t)$, where ω is the angular frequency, is suppressed.

The space dependence of the (partial) pressures in the fluids is given by the following expressions. For the pressure p_i of the incident wave

$$p_i = P_i \exp[-ik_0(x \sin\theta - z \cos\theta)], \quad (4)$$

for the pressure p_r of the reflected wave

$$p_r = P_r \exp[-ik_0(x \sin\theta + z \cos\theta)], \quad (5)$$

and for the pressure p_t of the transmitted wave

$$p_t = P_t \exp[-ik'(x \sin\theta' - z \cos\theta')]. \quad (6)$$

The total pressure in the fluid at the surface of the plate on the side of the incident wave is

$$p(z = d) = p_i + p_r = (P_0 + P) \exp(-ikx), \quad (7)$$

where $P_0 = P_i \exp(ik_0 d \cos \theta)$ and $P = P_r \exp(-ik_0 d \cos \theta)$. The pressure at the surface of the plate on the opposite side is

$$p(z = -d) = P' \exp(-ikx), \quad (8)$$

where $P' = P_t \exp(-ik'd \cos \theta')$. Here the coincidence condition is used, namely

$$k = k_0 \sin \theta = k' \sin \theta'. \quad (9)$$

The equations of motion in thick-plate theory are obtained by integration of the differential equations of motion in the direction perpendicular to the plate and by subsequent substitution of approximate average values for the integrals of the displacements and their moments. Details are given in Ref. [3]. There are two equations for the antisymmetric wave:

$$D \frac{\partial^2 \phi_x}{\partial x^2} - 2\kappa_1^2 G d \left(\frac{\partial W}{\partial x} + \phi_x \right) + \frac{2}{3} \rho_s \omega^2 d^3 \phi_x = 0, \quad (10)$$

$$\text{and} \quad 2\kappa_1^2 G d \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial \phi_x}{\partial x} \right) + q_z^a + 2\rho_s \omega^2 d W = 0, \quad (11)$$

and two equations for the symmetric wave:

$$(\lambda + 2G) \frac{\partial^2 U}{\partial x^2} + \lambda \frac{\partial \chi}{\partial x} + \rho_s \omega^2 U = 0 \quad (12)$$

$$\text{and} \quad \frac{2}{3} d^2 \kappa_2^2 G \frac{\partial^2 \chi}{\partial x^2} - 2(\lambda + 2G) \chi - 2\lambda \frac{\partial U}{\partial x} + q_z^s + \frac{2}{3} \rho_s \omega^2 d^2 \chi = 0. \quad (13)$$

The displacement components in the x direction parallel to the plate and in the z direction perpendicular to the plate are represented in thick-plate theory by

$$u = U + z \phi_x$$

$$\text{and} \quad w = W + z \chi, \quad (14)$$

respectively.

The symbols in Eqs. (10) through (14) are defined as follows. The elastic properties of the plate are characterized by Young's modulus E and Poisson's ratio ν , or alternatively by the first Lamé constant λ and the shear modulus G . The bending of a plate is characterized by a bending stiffness D equal to $(2/3)Ed^3/(1-\nu^2)$. These elastic constants are related to various wave speeds in the solid or in the plate. The speed c_d of dilatational waves in a solid is given by

$$\lambda + 2G = \rho_s c_d^2, \quad (15)$$

the speed c_s of shear waves in a solid is given by

$$G = \rho_s c_s^2, \quad (16)$$

and the speed c_p of extensional waves in a plate is given by

$$D = (2/3)d^3 \rho_s c_p^2. \quad (17)$$

The various wave speeds are normalized through division by c_s . The resulting relative wave speed is indicated by γ , which is subscripted in the same way as the corresponding wave speeds c . The displacements in the plate are described by an average displacement in the x direction U , an average displacement in the z direction W , an average rotational displacement of the cross section ϕ_x , and an average strain in the z direction χ . The correction factors κ_1^2 and κ_2^2 serve to relate the effective shear moduli connected with the thick-plate approximation to the actual shear modulus G . The loading of the plate by the fluids on both sides is expressed in terms of the normal stress σ_z on the faces of the plates. Because of the symmetry character of the two wave types, the loading of the plate for antisymmetric waves is given by a quantity q_z^a equal to $\sigma_z(d) - \sigma_z(-d)$, while the corresponding quantity for symmetric waves q_z^s is given by $q_z^s = \sigma_z(d) + \sigma_z(-d)$. The condition that the stress is continuous at any boundary leads to the following relations between the stress component σ_z and the pressure in the fluid at the two surfaces of the plate:

$$q_z^a = \sigma_z(z=d) - \sigma_z(z=-d) = -(P_0 + P) + P', \quad (18)$$

$$q_z^s = \sigma_z(z=d) + \sigma_z(z=-d) = -(P_0 + P) - P'. \quad (19)$$

Also, at the interfaces, the component of the particle velocity in the solid in the z direction has to equal that in the fluid. This leads to the relation between pressure gradient in the fluid and particle acceleration in the solid at a boundary. In general, one has

$$-\frac{1}{\rho_0} \frac{\partial p}{\partial z} = \frac{\partial^2 w}{\partial t^2} (\text{fluid}) = \frac{\partial^2 w}{\partial t^2} (\text{solid}), \quad (20)$$

which gives for $z = d$:

$$ik(P_0 - P)\cot\theta - \rho_0 \omega^2(W + \chi d) = 0, \quad (21)$$

and for $z = -d$:

$$ikP'\cot\theta' - \rho' \omega^2(W - \chi d) = 0. \quad (22)$$

If one assumes traveling wave solutions for the field variables ϕ_x , W/d , U/d , χ , the equations of motion, Eqs. (10)-(13) transform into linear algebraic equations. Adding the boundary conditions Eqs. (21) and (22) results in a set of six homogeneous equations in seven unknowns. This set can be advantageously represented in the form of a matrix of the coefficients. The rows are indicated by the numbers of the equations, and the columns are marked

by the corresponding amplitudes (see Table I). The pressure amplitudes are made dimensionless through division by $(\rho_0 c_s^2)$. By the familiar methods of linear algebra, one can derive the ratios between the amplitudes of the unknowns.

Table I Matrix of coefficients of equations describing reflection by fluid loaded plate.

Equations	Field variables \rightarrow					
	ϕ_x	W/d	U/d	x	$P_0/(\rho_0 c_s^2)$	$P/(\rho_0 c_s^2)$
(10)	$\frac{1}{3}(kd)^2(\gamma^2 - \kappa_1^2)$	$i(kd)\kappa_1^2$	0	0	0	0
(11)	$i(kd)\kappa_1^2$	$(kd)^2(\kappa_1^2 - \gamma^2)$	0	0	$\rho_0/2\rho_s$	$\rho_0/2\rho_s$
(12)	0	0	$(kd)^2(\gamma_d^2 - \gamma^2)$	$i(kd)(\gamma_d^2 - 2)$	0	0
(13)	0	0	$-i(kd)(\gamma_d^2 - 2)$	$\frac{1}{3}(kd)^2(\kappa_2^2 - \gamma^2) + \gamma_d^2$	$\rho_0/2\rho_s$	$\rho_0/2\rho_s$
(21)	0	$\gamma^2(kd)^2$	0	$\gamma^2(kd)^2$	$-i(kd)\cot\theta$	0
(22)	0	$\gamma^2(kd)^2$	0	$-\gamma^2(kd)^2$	0	$-i(kd)(\rho_0/\rho^*)\cot\theta$

REFLECTION OF SOUND BY PLATE LOADED BY FLUID ON ONE SIDE

The matrix of the coefficients of the equations describing reflection of sound by a plate loaded by a fluid on one side only is obtained from the matrix in Table I by omitting the sixth row and the seventh column. The matrix elements of the matrix of Table I are indicated by M_{ij} . Two subdeterminants are defined by the following expressions:

$$\Delta_a = M_{11} M_{22} - M_{12} M_{21} \quad (23)$$

and

$$\Delta_s = M_{33} M_{44} - M_{34} M_{43} \quad (24)$$

The reflection coefficient is evaluated first for the case where only antisymmetric waves participate. One finds that the factor (P/P_0) in the expression for the reflection coefficient given by

$$P_r/P_i = (P/P_0)\exp(2ik_0d\cos\theta) \quad (25)$$

is as follows:

$$(P/P_0)_a = \frac{2i\Delta_a \cot\theta + (kd)\gamma^2(\rho_0/\rho_s)M_{11}}{2i\Delta_a \cot\theta - (kd)\gamma^2(\rho_0/\rho_s)M_{11}} \quad (26)$$

The subscript a refers to antisymmetric waves. By algebraic manipulation one can transform the right-hand side of Eq. (26) to

$$(P/P_0)_a = 1 - \frac{2}{1 - (2i\Delta_a \cot\theta)/[(\rho_0/\rho_s)\gamma^2 kd M_{11}]} \quad (27)$$

This form of the expression shows the appearance of the structural response function Ω_a for antisymmetric waves defined by

$$\Omega_a = - \frac{2\Delta_a}{(\rho_0/\rho_s)\gamma^2 kd M_{11} \sin\theta} \quad (28)$$

In an analogous manner, one can show that the pertinent factor in the reflection coefficient for symmetric waves only may be written as

$$(P/P_0)_s = 1 - \frac{2}{1 + i\Omega_s \cos\theta} \quad (29)$$

where the structural response function Ω_s for symmetric waves is defined by

$$\Omega_s = - \frac{2\Delta_s}{(\rho_0/\rho_s)\gamma^2 kd M_{33} \sin\theta} \quad (30)$$

The subscript s refers to symmetric waves.

If one allows both antisymmetric and symmetric waves to participate, the first factor in the expression for the reflection coefficient becomes

$$(P/P_0) = \frac{2i(\cot\theta)\Delta_a\Delta_s + \gamma^2 kd(\rho_0/\rho_s)M_{33}\Delta_a + \gamma^2(kd)(\rho_0/\rho_s)M_{11}\Delta_s}{2i(\cot\theta)\Delta_a\Delta_s - \gamma^2(kd)(\rho_0/\rho_s)M_{33}\Delta_a - \gamma^2(kd)(\rho_0/\rho_s)M_{11}\Delta_s} \quad (31)$$

This can be again written in terms of a response function Ω for the combination of the two types of waves,

$$(P/P_0) = 1 - \frac{2}{1 + i\Omega \cos\theta} \quad (32)$$

where the response function Ω is related to the response functions for

antisymmetric and symmetric waves separately by

$$\frac{1}{\Omega} = \frac{1}{\Omega_a} + \frac{1}{\Omega_s} . \quad (33)$$

This relation concurs with the interpretation of the response function as describing the displacement in the solid as a response to the pressure exerted at the face of the plate. The two types of waves act as two "channels", and the response function for the two channels in parallel is computed as one would in the case of an equivalent impedance for two parallel impedances.

Notice that the absolute value of the reflection coefficient is unity, the reflection causes only a phase shift in the wave. This is understandable since no energy is lost in the process, due to the absence of an energy-dissipating mechanism.

RELATIVE CONTRIBUTION OF ANTISYMMETRIC AND SYMMETRIC WAVES

For the case of reflection or transmission of sound by a plate, one can assess the relative participation of the two types of waves by looking at the ratios of the displacement components. The following ratio of the displacement U in the x direction due to the symmetric wave divided by the corresponding displacement due to the antisymmetric wave $\phi_x d$ at the surface of the plate is obtained from the matrix in Table I,

$$U/(\phi_x d) = \frac{M_{34}\Delta_a}{M_{12}\Delta_s} . \quad (34)$$

The analogous expression for the displacements in the z direction is

$$x d/W = \frac{M_{33}\Delta_a}{M_{11}\Delta_s} . \quad (35)$$

To obtain a measure for the relative participation of the two wave types based on both u and w displacement components, one might consider the kinetic energy density E averaged over a cross section and one cycle. The average energy density E for each wave is proportional to the cross-sectional average of the sum of the squares of the u and w displacement components of that wave; i.e.,

$$E \propto 1/d \int_0^d (u^2 + w^2) dz . \quad (36)$$

The ratio of the average kinetic energy density of the symmetric wave E_s to that of the antisymmetric wave E_a is found to be

$$\frac{E_s}{E_a} = \frac{(U/d)^2 + \chi^2/3}{(W/d)^2 + \phi_x^2/3} = \left(\frac{\Delta_a}{\Delta_s} \right)^2 \frac{M_{34}^2 + M_{33}^2/3}{M_{11}^2 + M_{12}^2/3} . \quad (37)$$

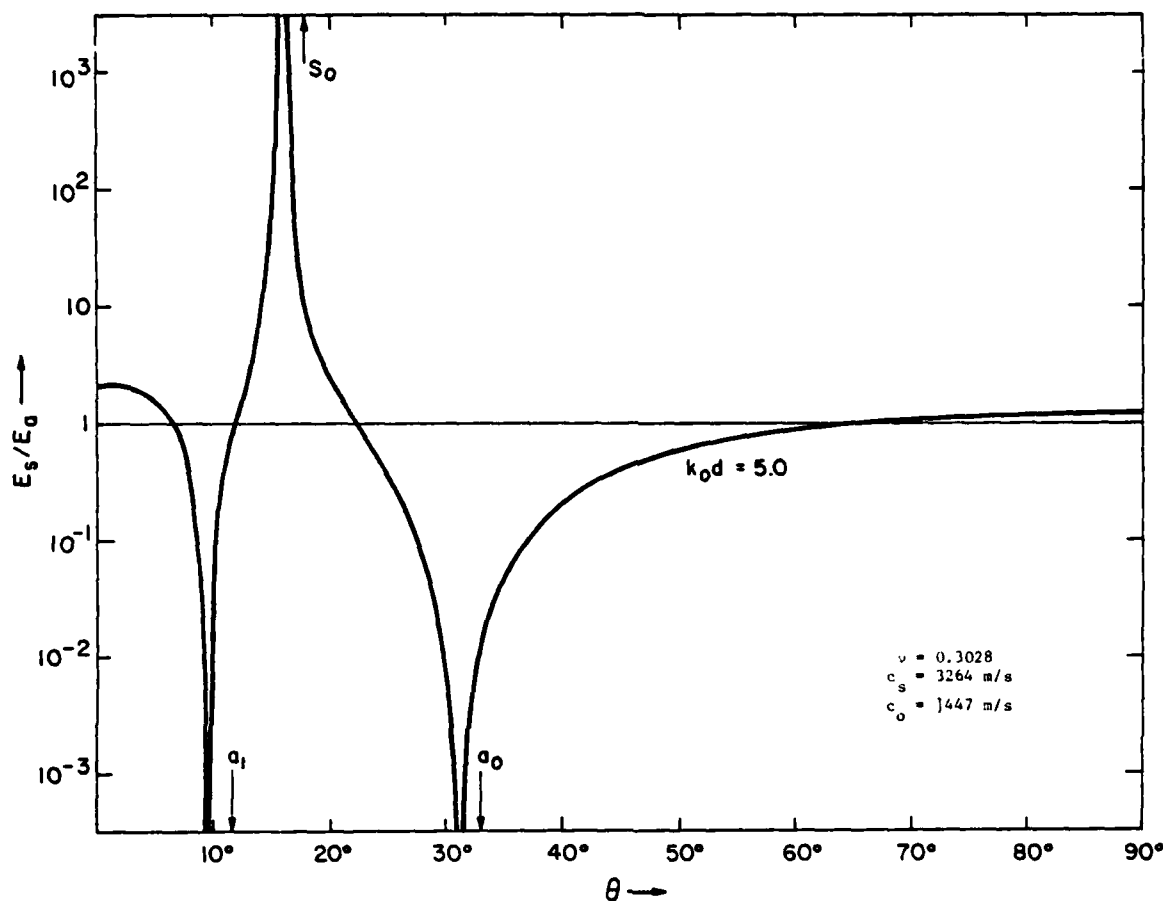


Fig. 2 - Relative participation of symmetric and antisymmetric waves in reflection by plate

In Fig. 2, the ratio E_s/E_a from Eq. (37) is graphed as a function of the incidence angle θ . The specific case of steel with $\nu = 0.3028$ and $c_s = 3264$ m/s is chosen. With a propagation speed of sound in water at 4°C of 1447 m/s, the value of c_o/c_s is equal to 0.443 . The value of the dimensionless wave number was chosen to be $k_0 d = 5$. The correction factors κ_1 and κ_2 in the matrix elements of Table I used in Eq. (37) are set equal to γ_R the relative Rayleigh wave speed. The resonance type features in Fig. 2 are due to coincidence between the trace speed of the incident wave and the free wave speed of antisymmetric or symmetric waves in the plate. The specific type of the free wave is indicated by the symbols a and s , respectively. The subscripts o and 1 indicate the order of the wave. One may notice that away from these coincidences the kinetic energy is practically evenly divided between the antisymmetric and the symmetric waves.

The behavior of the graph in Fig. 2, in relation to the material properties of the fluid and plate, is clarified by Fig. 3. In this figure, the dispersion relations for Lamb waves are shown in dimensionless form, namely the relative wave speed $\gamma = c/c_s$ as a function of the non-dimensional wave number $k_s d$ for shear waves. The only free parameter in this figure is Poisson's ratio ν . It determines the exact shape of the curves, but qualitatively the general behavior of the curves varies little when Poisson's ratio is given a different value. The dispersion curves are shown for the zero and first-order antisymmetric Lamb waves a_0 and a_1 , respectively, and for the zero and first-order symmetric Lamb waves s_0 and s_1 , respectively. This figure is intended for qualitative information only; it is only approximately correct in its scale.

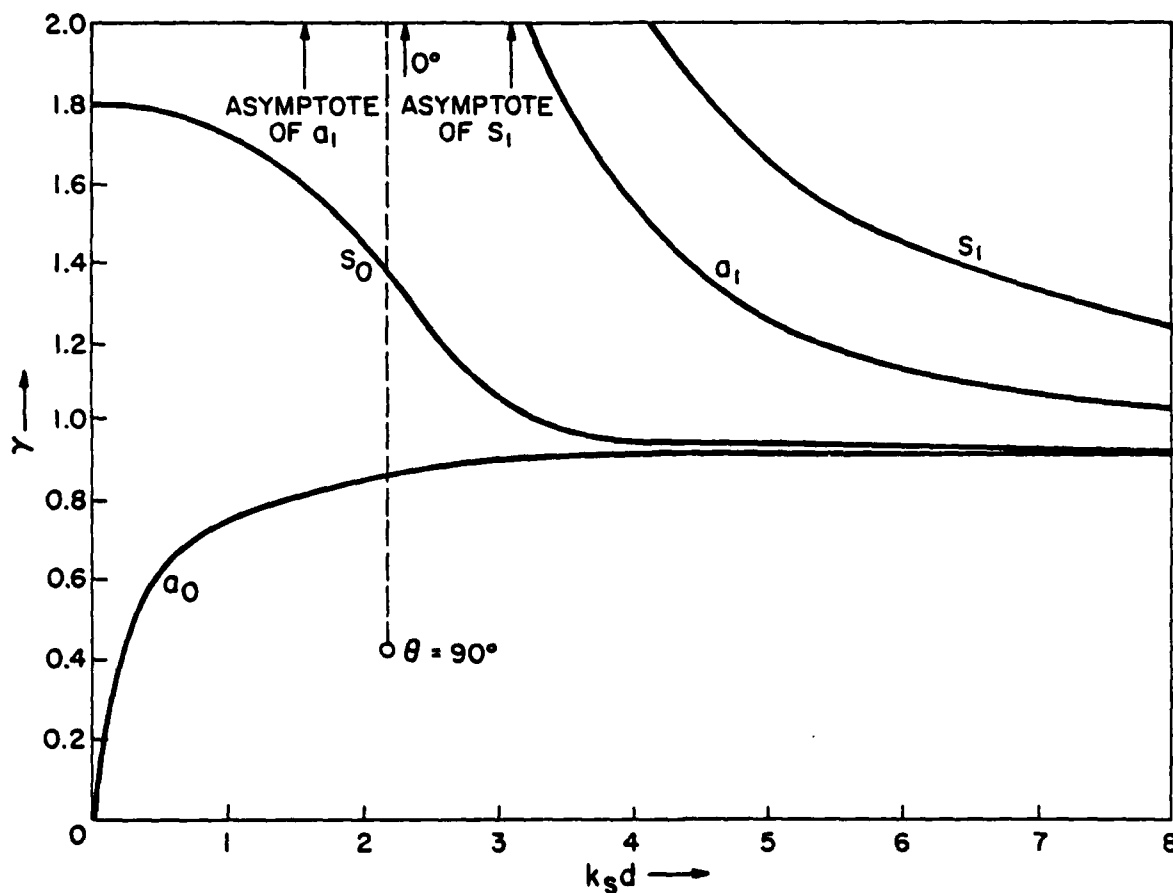


Fig. 3 - Graphic representation of coincidence condition

A wave impinging on the plate is characterized by both a wave number $k_0 d$ in the fluid and by the angle of incidence θ . The coincidence

condition given by Eq. (9) fixes the relative phase speed γ in the plate for a given value of $k_0 d$ and a given angle θ by the relation

$$\gamma = c_0 / (c_s \sin \theta). \quad (38)$$

The graphic representation of this equation is shown in Fig. 3 by the dashed line parallel to the vertical axis. This line starts at $\theta = 90^\circ$ and goes to infinity for $\theta \rightarrow 0^\circ$. The location of the starting point at $\theta = 90^\circ$ is determined by the ratio of the phase speed in the fluid to the shear wave speed in the plate c_0/c_s . The abscissa of the starting point is $k_{sd} = (k_0 d) / (c_0/c_s)$, and the ordinate is $\gamma = c_0/c_s$. Wherever the dashed line in Fig. 3 intersects a dispersion curve, the trace speed of the incident wave equals the free wave speed of one of the wave types. That wave type will be dominant due to the presence of the factor $(\Delta_a/\Delta_s)^2$ in Eq. (37). The location of the starting point at $\theta = 90^\circ$ determines which of the four dispersion curves will be intersected by the dashed line. Thus the value of the wave number $k_0 d$ in the fluid and the ratio of the wave speed in the fluid to the shear wave speed in the solid c_0/c_s determine the appearance of the curve in Fig. 2. This explains why, in this specific figure, one has coincidence peaks for the three wave types a_0 , s_0 , and a_1 , respectively, starting from $\theta = 90^\circ$.

CONCLUSIONS

The main conclusion of the present study is that one should take into account the contribution of symmetric waves to the reflection of sound by a plate. Coincidence between the trace speed of the impinging wave in the fluid and the free wave speed of a wave in the plate of a given type, antisymmetric or symmetric, leads to dominance of that wave type over the other. Away from coincidence, the distribution of kinetic energy between the two wave types is about equal.

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